Measuring the angle-dependent sound absorption coefficient with a small microphone array

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Introduction

Measurement methods for the measurement of the oblique incidence sound absorption coefficient generally rely upon an overall model of the acoustic field. Such models typically describe plane- or spherical wave incidence upon a planar, locally-reacting, surface of infinite extent in a semi-free field. However, in general, the actual acoustic field deviates from the model. Possible causes for deviations are:

- The source directivity is different from the assumed directivity
- The local reaction assumption typically breaks down for large angles of incidence
- The edges of the sample lead to edge-diffracted waves
- Room reflections may be present, in particular at low frequencies

If such field deviations are present, fitting the measurement results to the model may result in a sound absorption coefficient that is inaccurate to a greater or lesser extent. Therefore, we have developed a novel method that relies upon a local field assumption only, so that less requirements for the acoustic field are to be fulfilled. By doing so, measurements may be performed in non-ideal sound fields without having knowledge of the source directivity and of the physical behavior of the sample.

This novel method relies upon a Local Specular Wave Assumption and is therefore called the LSPW-method. The theory of this method is elucidated in the following section, presenting an approach for laboratory measurements as well as for in situ measurements. Results of both approaches are compared for a sound absorbing panel.

Theory

In the following, we assume linear acoustics and stationary pure tone sound fields. I.e. the Helmholtz-equation can be used. Here the $\exp(i\omega t)$-convention is used. Vector quantities are bold-faced, and capitalized letters refer to quantities in the frequency-domain.

The local field assumption is illustrated in Fig. 1, showing the sample surface S with a measurement surface S_p at distance d. In each point upon S_p, we assume that the acoustic field can be approximated by two plane waves, one incident wave (A), and one specularly reflected wave (B). Variations of the phases, amplitudes and directions of both waves with space are explicitly allowed. The procedure with which the sound absorption coefficient is determined depends on what type of measurement is performed. One can now distinguish between a) laboratory measurements, where the angle of incidence is known, and b) in situ measurements, where the effective angle of incidence is generally unknown. Each procedure will be presented in a separate section.

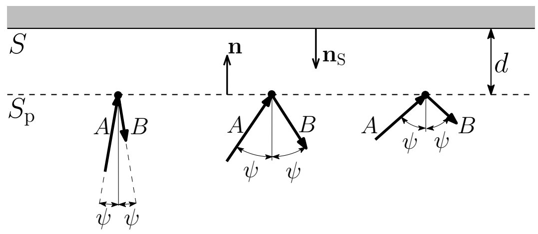

Figure 1: Approximation of an acoustic field locally by a one incident (A) and one reflected wave (B).

Laboratory measurement

In this case, it is assumed that the measurement is performed in a free- or semi-free field, so that the angle of incidence is known in advance. In this case, one needs to determine the complex amplitudes A and B of the incident- and reflected wave. To obtain these amplitudes one has to measure two independent quantities. One may use two acoustic pressure signals (pp), two particle velocity signals (uu) or combination of both (pu) for this purpose. Here, the procedure for two acoustic pressures is elucidated.

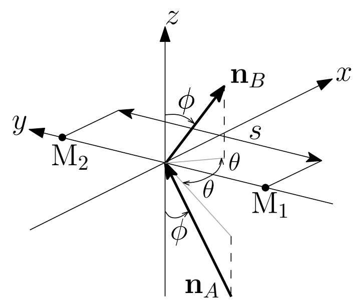

Figure 2: Direction convention in 3 dimensions. M_1 and M_2 indicate the positions of microphone 1 and 2.

Considering the direction convention shown in Fig. 2, one can write the acoustic pressures at both microphones M_1 and M_2 in terms of A and B. Equating these expressions to the measured complex acoustic pressures, one obtains the following two equations:

$$\begin{bmatrix} e^{ik\frac{s}{2}\cos\psi} & e^{-ik\frac{s}{2}\cos\psi} \\ e^{-ik\frac{s}{2}\cos\psi} & e^{ik\frac{s}{2}\cos\psi} \end{bmatrix} \begin{bmatrix} A(\mathbf{r}) \\ B(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} P_1(\mathbf{r}) \\ P_2(\mathbf{r}) \end{bmatrix}, \tag{1}$$

where, $\mathbf{r}$ is a vector defining the position of the measurement point in an overall coordinate system, $\psi = \arccos(\sin\phi\cos\theta)$ is the angle of incidence, k the (real) wavenumber, and s the microphone spacing. $A(\mathbf{r})$ and $B(\mathbf{r})$ can straightforwardly be determined by solving these two equations. Once these are known, one can calculate the incident acoustic intensity in direction $\mathbf{n}$ and the reflected acoustic intensity in direction $-\mathbf{n}$ with:

$$I_{\text{in}}(\mathbf{r}) = \frac{|A(\mathbf{r})|^2}{2\rho_0 c_0}\cos\psi, \tag{2}$$

$$I_{\text{refl}}(\mathbf{r}) = \frac{|B(\mathbf{r})|^2}{2\rho_0 c_0}\cos\psi, \tag{3}$$

where ρ_0 and c_0 are the mass density and speed of sound, respectively. As we are approximating the field locally with plane waves, the active acoustic intensity can be calculated by subtraction of the reflected- from the incident acoustic intensity:

$$I_{\text{ac}}(\mathbf{r}) = \frac{|A(\mathbf{r})|^2 - |B(\mathbf{r})|^2}{2Z_0}\cos\psi. \tag{4}$$

Knowing the local active- and incident acoustic intensity, one can calculate the local sound absorption coefficient To determine the area-averaged sound absorption coefficient, one can spatially integrate both intensities over the area of interest according to

$$W_{\text{ac}}(\tilde{\psi}) = \int_{S_{\text{p}}} I_{\text{ac}}(\mathbf{r})\,\mathrm{d}S, \tag{5}$$

$$W_{\text{in}}(\tilde{\psi}) = \int_{S_{\text{p}}} I_{\text{in}}(\mathbf{r})\,\mathrm{d}S \tag{6}$$

where $\tilde{\psi}$ is the area-averaged angle of incidence. The effective, area-averaged, oblique incidence sound absorption coefficient for surface S_{p} can be calculated by taking the ratio of the active- and incident acoustic power

$$\alpha(\tilde{\psi}) = \frac{W_{\text{ac}}(\tilde{\psi})}{W_{\text{in}}(\tilde{\psi})}, \tag{7}$$

α is an *effective* sound absorption coefficient as the angle of incidence typically varies over the measurement surface when a spatially fixed source is used. To obtain the area-averaged oblique incidence sound absorption coefficient $\alpha(\psi)$ for a well-defined angle of incidence ψ, one must either realize plane wave incidence over the whole surface area of S_{p}, or determine the active- and incident acoustic power associated with a sub-area of S_{p}, at which the angle of incidence lies within small tolerances of the desired angle of incidence.

The measurement of the active acoustic intensity can be performed using an unidirectional sound intensity probe, typically being either a pu-probe [4], or a pp-probe [5].

The measurements discussed in this paper, were performed with a newly developed 3D sound intensity probe, consisting of 8 digital MEMS-microphones [1], see Fig 3.

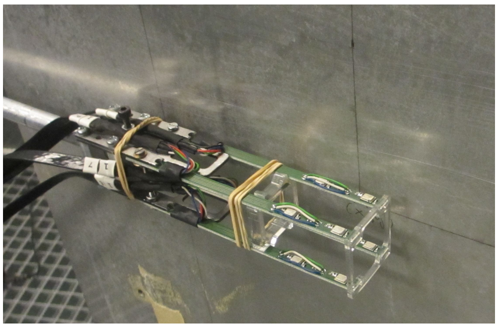

Figure 3: 3D sound intensity probe in front of a plate.

Defining the measurement point as the geometric center of the probe, one can alternatively define the problem of determining $A(\mathbf{r})$ and $B(\mathbf{r})$ in terms of a least-squares problem:

$$\mathbf{Mv} = \mathbf{P} \tag{8}$$

in which $\mathbf{v}$ is the vector containing $A(\mathbf{r})$ and $B(\mathbf{r})$, $\mathbf{P}$ contains 8 complex acoustic pressures, and the columns of matrix $\mathbf{M}$ are given by:

$$M_{j1} = e^{-ik[x_j\sin\phi\sin\theta + y_j\sin\phi\cos\theta + z_j\cos\phi]},$$
$$M_{j2} = e^{-ik[x_j\sin\phi\sin\theta - y_j\sin\phi\cos\theta + z_j\cos\phi]}, \tag{9}$$

where x_j, y_j, and z_j are the spatial coordinates of microphone j relative to the geometric center of the probe, acc. the coordinate system shown in Fig. 2. One can solve for $\mathbf{v}$ in a least-squares sense to obtain:

$$\mathbf{v} = \left(\mathbf{M}^{\text{H}}\mathbf{M}\right)^{-1}\mathbf{M}^{\text{H}}\mathbf{P}. \tag{10}$$

where the superscript $^{\text{H}}$ denotes the Hermitian transpose. By doing so, the sum of all squared errors is minimized.

In situ measurement

In this case, the angle of incidence is generally unknown, and has to be determined by the measurement. The system of equations (8) is the same, but is non-linear, as one has to solve for the angles ϕ and θ, see Fig. 2, as well. Accordingly, a non-linear solving procedure has to be employed. We have used the multi-dimensional Newton-Raphson method to this purpose. First, we define the vector of unknowns for iteration j:

$$\mathbf{g}_j = [\ A \quad B \quad \phi \quad \theta\ ]_j^T. \tag{11}$$

The residual of the system of equations associated with this vector is $\mathbf{F}_j$, defined as:

$$\mathbf{F}_j = \mathbf{M}_j \mathbf{v}_j - \mathbf{P}. \tag{12}$$

The next iteration step is then performed using the next guess for $\mathbf{g}$:

$$\mathbf{g}_{j+1} = \mathbf{g}_j - \mathbf{J}_j^{-1} \mathbf{F}_j, \tag{13}$$

where $\mathbf{J}_j$ is the Jacobian of the matrix $\mathbf{M}$, which can be calculated analytically for each given value of ψ and θ. The iteration procedure is stopped if a) the change of the value of the functional $G_j = \mathbf{F}_j^{\mathrm{H}} \mathbf{F}_j$ falls below a certain defined value, or b) if a pre-defined maximum number of iterations is exceeded.

A typical issue associated with the use of a non-linear equation solver is the dependency of the results on the choice of the start vector used in the first iteration. Although the effective angle of incidence may be unknown in an in situ measurement, one usually can provide an estimate that is sufficiently accurate for the solver to converge.

Results

Measurements were performed for a sound absorbing panel. This panel consists of a perforated cover plate, an air gap completely filled with a 50 mm thick sheet of mineral wool, and a rigid backing. Area-averaging was performed using a measurement grid of 11x11 points over a surface area of 128x128 mm^2. The geometric center of the probe was located at a distance of 30 mm to the surface of the panel. The probe was moved from point to point using a PC-controlled scanning system with a positioning accuracy of less than 1 mm. At each point, a recording of 10 s was carried out. A loudspeaker with a spherical housing is used as the sound source. It is positioned at a distance of 1 m from the center of the area of interest.

Figure 4 shows the area-averaged oblique incidence sound absorption coefficient obtained with the LSPW-method and the 1/3-octave values obtained with a measurement in a reverberation room acc. ISO 354 [6] as provided by the manufacturer of the panel.

At the helmholtz-resonance between 500 and 1000 Hz, the sound absorption coefficient is almost independent of the angle of incidence up to 45°. Sound absorption is clearly higher 60° incidence. The measurement was performed in a large well-absorbing, but not anechoic, room. Although room reflections were present, and the area of the measurement surface was very small, credible curves for the sound absorption coefficient are obtained with the LSPW-method. Although a measurement for 75° incidence could not be performed it is not likely that

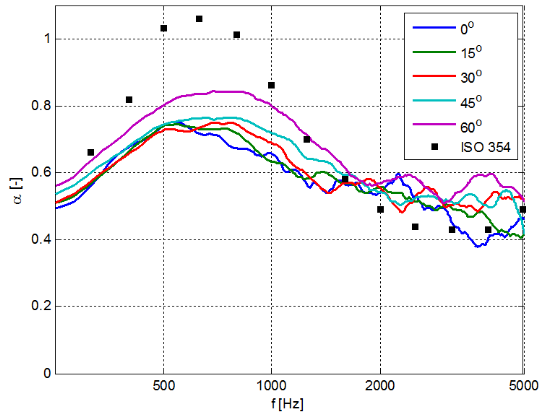

Figure 4: Area-averaged sound absorption coefficient for different angles of incidence obtained with the LSPW-method.

this measurement would explain the difference between the values obtained with the reverberation room method and the LSPW-method. Instead, it is pointed out that the LSPW-method tends to overestimate the sound absorption coefficient if the incident waves have a diverging propagation path, because the method does not account for geometrical spreading between the measurement- and the material surface.

One could consider compensating for geometrical spreading, however, we explicitly do not want to extrapolate the incident- and reflected wave to the surface of the absorbing panel as such an extrapolation would require knowledge of the directivity of the source as well as knowledge about wave propagation inside the panel. Instead, the source should be located at a sufficiently large distance from the sample. However, for the measurements discussed here, the setup did not allow for a source distance significantly larger than 1 m.

Figure 5 shows a comparison of the sound absorption coefficient for 45° incidence obtained with the linear (least-squares) procedure and the non-linear solving procedure. Both curves match in a qualitative sense, but the curve obtained with the non-linear solving procedure is more irregular. These irregularities are related to variations of the curves of the angles ϕ and θ in Fig. 6, showing both angles as a function of frequency for all 121 measurement points.

The estimate for the angle ϕ in Fig. 6 is not exactly equal to 90° but lies in a small zone around this angle. This is possible, as the source remained at a fixed position during the measurements, so that the angle of incidence indeed varied from point to point. The estimate for the angle θ varies more strongly and deviates somewhat more. By comparing Fig. 5 with Fig. 6, it can be noticed that deviations in the sound absorption coefficient coincide with deviations in the curve of θ. The found angles of θ seem to vary too much given the size of the measurement area and a source distance of 1 m. These variations may be correct, if they are induced by room reflections, leading to a change of the effective angle of incidence. However, they also may be the result of near-field effects

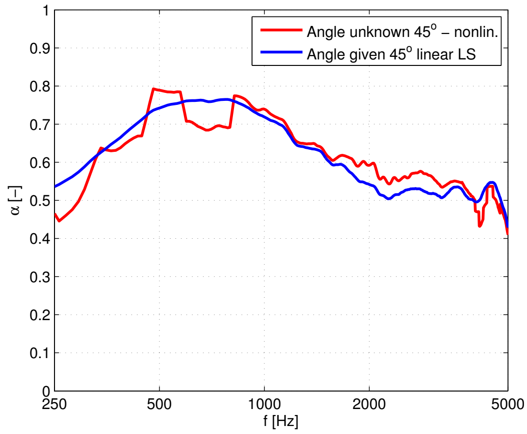

Figure 5: Area-averaged sound absorption coefficient for 45° incidence obtained with the linear least-squares and the non-linear Newton-Raphson solving procedure.

as the nearest microphones were only located 10 mm from the absorber's surface.

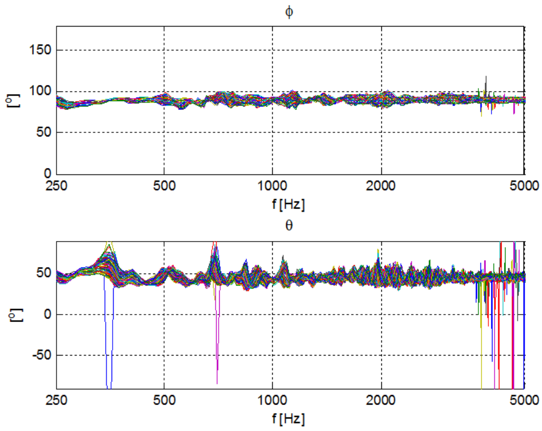

Figure 6: Angles determined with the non-linear LSPW-method for 45° incidence.

Analysis of the results for the non-linear solving procedure for angles greater than 50° showed that the accuracy of the estimated angles decreases and the sound absorption coefficient curve becomes more and more distorted as the angle of incidence increases. To improve the robustness of this procedure, it is proposed to increase the microphone spacing for large angles of incidence, thus reducing the sensitivity to phase-mismatch of the microphones. In addition, to avoid near-field effects, it is useful to increase the distance of the probe to the surface of the absorber. As the LSPW-method does not use a model for the propagation of acoustic waves between the measurement surface and the surface of the sample, the distance of the source to the measurement surface shall be sufficiently large to nearly obtain plane wave incidence.

Conclusions

The LSPW-method is a novel method for the measurement of the area-averaged oblique incidence sound absorption coefficient. It relies upon a so-called local specular plane wave assumption, and does not require an overall model of the acoustic field. Accordingly, knowledge about the source directivity and the character of the sample are not required. The concept of area-averaging makes the method also suitable for measurement in non-ideal acoustic fields. Results of measurements for a sound absorbing panel show that the method can be applied for laboratory measurements, but also has potential for in situ measurements. For the latter case, it is recommended to perform further investigations in order to further improve the robustness of the non-linear solving procedure. Finally, it is desirable to carry out further research with respect to measurements in near-fields.

Acknowledgements

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